On the interior scattering of waves, defined by hyperbolic variational principles

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Dedicated to I.M. Gelfand
on his 75th birthday

Abstract. *The transformation of waves of different kinds at some interior points of nonhomogeneous media is studied for waves defined by linear hyperbolic variational systems.*

A formal normal form is given for the light hypersurface (i. e. for the dispersion relation) at its generic singular points in terms of the contact geometry. This normal form describes the behaviour of the rays and of the wavefronts at the singular points corresponding to the multiple eigenvalues of the principal symbol of a generic hyperbolic variational problem.

INTRODUCTION

The word «waves» below always means «waves, defined by linear equations». In the linear theory the waves of different kinds usually propagate independently. However in nonhomogeneous media the transformation of waves of different kinds becomes typical for some interior points of the domain. This unusual interior scattering is studied in the present article at the level of the geometrical optics for the waves, described by hyperbolic variational principles. The main result is a formal normal form for the characteristics of the interior scattering in typical multidimensional variational hyperbolic linear systems.

Key-Words: Hyperbolic system, variational principle, singular points, nonstrict hyperbolicity; wave fronts, rays, contact geometry, multiple eigenvalues, normal form.
1980 MSC: 35, 49, 53, 58, 73, 78, 81, 86.

Mathematically, the problem is reduced to the study of singularities of the light hypersurface in a contact manifold. To explain this mathematical problem I recall first some elements of contact geometry, which plays in the theory of propagation of waves the role played by symplectic geometry in mechanics.

A *contact structure* on an odd-dimensional manifold is a distribution of tangent hyperplanes which is maximally nondegenrate at every point. All such distributions (of any given dimension) are locally diffeomorphic (the Darboux theorem). The main contact manifold of the theory of wave propagation is the space PT^*V of all contact elements of the space-time V.

A *contact element* on a manifold is a hyperplane of the tangent space. The space of contact elements of a manifold is the projectivization of its tangent bundle. The fiber of this projectivized bundle is the projective space of all the contact elements, tangent to the manifold at one point of contact.

The contact structure in the space of contact elements is defined by the following condition: the velocity of a moving contact element belongs to the hyperplane of the distribution, if and only if the contact points velocity belongs to the moving contact element.

The dimension of the space of contact elements of a space-time of dimension $D+1$ is equal to $2D+1$. The rays and the propagation of the waves are defined by some «light hypersurface» in this contact manifold.

Any hypersurface in a contact manifold is intrinsically equipped with its field of characteristic directions. If the contact structure is (locally) defined as the distribution of the zeroes of a 1-form, $\alpha = 0$, the *characteristic direction* at a point of the hypersurface is defined as the symplectic orho-complement to the intersection of the tangent space of the hypersurface and the contact plane $\alpha = 0$, whose symplectic structure is defined by $d\alpha$. The integral curves of the field of characteristic directions are called the *characteristics* of the hypersurface. Their projections to the space-time are the *rays.* The maximal integral manifolds of the contact structure, contained in the light hypersurface, are called its *Legendre submanifolds*, their projections to the space-time are called *big wave fronts*, the sections of these by the isochrones - the *momentary wave fronts.*

The main result of the present work states that *the light surface of a typical variational hyperbolic system is reduced, in a neighbourhood of a typical singular point, to the microlocal formal normal form* $H = 0$, *where* $H = p_1^2 \pm q_1^2 - q_2^2$, *for the some local Darboux coordinates (i. e. coordinates such that the contact structure is defined by the equation* $\alpha = 0$, *where*

$$\alpha = dz + (p\,dq - q\,dp)/2, \; z \in R, p \in R^{D}, q \in R^{D}, D > 1.$$

The one-dimensional medium case ($D = 1$) is studied in [1]. In this case the normal form is more complicated:

$$H = p^2 \pm q^2 - z^2 + Cz^3.$$

Our normal forms allows us to find the geometry of the rays and wave fronts, as well as that of their perestroikas; for $D = 1$ the answers are described in [1]. For instance, the equation for the characteristics of the light surface reduced to the normal form is easily solved $(\dot{q}_1 = p_1, \dot{p}_1 = \pm q_1, \dot{p}_2 = q_2; \dot{q}_2 = \ldots = \dot{q}_D = \dot{p}_3 = \ldots = \dot{p}_D = \dot{z} = 0)$. For $D = 2$ a Legendre submanifold is just any oneparametric family of the characteristics. This information suffices for the study of the geometry of wave-fronts, but in this paper only the normal form of light hypersurface's singularities will be discussed.

According to [1], any $2D$-dimensional light hypersurface of a typical variational system is locally diffeomorphic to the quadratic cone $u^2 + v^2 = w^2$ in $R^{2D} + 1$ in some neighbourhood of a typical singular point of the hypersurface (1). Hence below I discuss only the reduction to the normal form of a contact structure given in a neighbourhood of a vertex of this cone.

REMARK 1. The abovementioned result of [1] implies an important topological difference between the general theory of hyperbolic systems and that of variational hyperbolic systems. Indeed, the nonstrict hyperbolicity points (i. e. the singularities of the light hypersurface) are encounteed, in typical general hyperbolic equations, only at the boundary of the hyperbolicity domain.

In contrast with this, the variational hyperbolic systems admit points of nonstrict hyperbolicity as a rule rather than as an exception: the light hypersurface of a generic variational hyperbolic system has singular points inside the hyperbolicity domain (they form a codimension 2 subset of the light hypersurface).

This leads to some topological problems.

Example. Consider the space $F = R^{m(m+1)/2}$ of quadratic forms in m variables. We call a polynomial mapping $f : R \to F$ of degree d a *hyperbolic mapping* if the equation $\det f(t) = 0$ has md real roots (counted with their multiplicities). All such mappings form a space which we denote $\Gamma(m, d)$.

A variational hyperbolic system over the space-time V of dimension $D + 1$ defines (at any point of V) a sphere mapping $S^{d-1} \to \Gamma(m, d)$. Hence the following problems arise: 1) calculate $\pi_i(\Gamma(m, d)$;

2) which elements of these groups are represented by hyperbolic variational (pseudo?) differential systems?

3) find the connected components of the space of hyperbolic variational systems having fixed (m, D, d).

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(1) Strictly speaking, [1] deals with a special case, where the time is explicitly separated, but the proof still holds for the general case (with some small variations, for which I am indebted to B. A. Hessin).

There exist similar problems for the bundles over V, but those are material only in the cases of non-trivial answers to the preceding problems $1-3$.

REMARK 2. The series which reduce an analytic light surface to its normal form are generically divergent in the one-dimensional media case $D = 1$ (see [1]).

There is no divergence proof for the multidimensional case $D > 1$. For $D = 2$ the form $d\alpha$, lifted to the 2-fold covering of the cone, has the Martinet [2] singularity.

1. THE NORMAL FORMS

THEOREM *Any generical variety, diffeomorphic to the cone* $u^2 + v^2 = w^2$ *in the space* $R^{2D+1}(D > 1)$, *equipped with a contact structure* $\alpha = 0$, *is reducible by a formal local diffeormorphism to the normal form* $H = 0$, *where*

$$H = p_1^2 \pm q_1^2 - q_2^2,$$

(1) $$\alpha = dz + (p\,dq - q\,dp)/2, \; p = (p_1, \dots, p_D), \; q = (q_1, \dots, q_D). \quad \blacksquare$$

The genericity conditions (A and B) are explicitly stated below.

Let us consider the manifold of the vertices of the cone. It is defined by the equation $u = v = w = 0$ and has codimension 3.

DEFINITION. A submanifold of a manifold with a contact structure $\alpha = 0$ is *nondegenerate*, if $\alpha \wedge (d\alpha)^m \neq 0$, where the dimension of the submanifold is equal to $2m + 1$ or $2m + 2$.

LEMMA 1. *A nondegenerate submanifold of codimension* 3 *is reducible by a contactomorphism* (= a *diffeomorphism, preserving the contact structure defined by equation* (1)) *to a local normal form which is the codimension* 3 *coordinate plane* $p_1 = q_1 = q_2 = 0$ (*for codimension* $2r + 1 > 1$ *submanifolds the answer is* $p_1 = \dots = p_r = q_1 = \dots = q_{r+1} = 0$).

Proof. The classical theory of Pfaff equations implies that the restriction of the equation $\alpha = 0$ to a nondegenerate submanifold of dimension $2m + 2$ is locally reducible to the normal form,

$$dz + (p\,dq - q\,dp)/2 = 0, \; p = (p_1, \dots, p_m), \; q = (q_1, \dots, q_m).$$

The restriction of the 1-form α, given by (1), to the coordinate plane described in Lemma 1 has the same form.

But the restriction of the equation $\alpha = 0$ to any submanifold of a contact space defines the submanifold locally up to a contactomorphism (this is the Darboux - Givental theorem, see [3] ch. 4 n° 1.3 or [1] § n°4). Hence any nondegenerate submanifold is locally contactomorphic to the coordinate plane, described in Lemma 1, which is thus proved. ∎

For generic contact structures in the space containing the cone $u^2 + v^2 + w^2$ the following condition holds:

CONDITION A. *The manifold of vertices is nondegenerate at the point 0.*

From now on we suppose that this condition is satisfied. Let us choose the local Darboux coordinates for which the manifold of vertices of our hypersurface $H = 0$ has the form $p_1 = q_1 = q_2 = 0$. Such coordinates exist by Lemma 1.

DEFINITION. A (smooth, analytic...) function is *admissible*, if it has, at each point of the manifold of vertices $p_1 = q_1 = q_2 = 0$, a zero of order 2 (or greater).

Any admissible function admits a representation in the form of a quadratic form of (p_1, q_1, q_2), whose coefficients are smooth (analytic...) functions of all the Darboux coordinates (z, p, q). Our variety $H = 0$ is defined by an admissible function $H = u^2 + v^2 - w^2$.

LEMMA 2. *The tangent cone of the hypersurface $H = 0$ at its vertex 0 is given by the equation $F(p_1, q_1, q_2) = 0$, where F is a nondegenerate quadratic form of three variables.*

Proof. To obtain the quadratic form F one evaluates the coefficients of the quadratic form with variable coefficients, described above, at the point 0. ∎

For generic contact structures in the space, containing the hypersurface $H = 0$, the following holds.

CONDITION B. *The restriction of the form F to the plane $q_2 = 0$ is nondegenerate.*

REMARK. While this condition is formulated using the coordinates, it has an intrinsic meaning. The 3-space with coordinates (p_1, q_1, q_2) is the normal space to the manifold of vertices at the pont 0 (the quotient of the tangent space at 0 of the ambient space by its subspace tangent to the manifold of vertices).

This quotient inherits a Poisson structure form the symplectic structure of the contact distribution hyperplane of the ambient space. The symplectic leaves of this Poisson structure are $q_2 = \text{const}$.

DEFINITION. A diffeomorphism is *admissible* if it preserves the manifold of vertices $p_1 = q_1 = q_2 = 0$.

The admissible diffeomorphisms transform the admissible functions into admissible ones.

LEMMA 3. *Let H be an admissible function, satisfying the nondegeneracy condition B; then there exists an admissible contactomorphism, preserving 0 and reducing H to the form $H = H_2 + R$, where $H_2 = C(p_1^2 \pm q_1^2 - q_2^2)$ and where R is an admissible function having a zero of order 3 (or greater) at the origine.*

Proof. Any linear symplectic (preserving the 2-form $dp \wedge dq$) tranformation $(p,q) \mapsto (P,Q)$ generates a contactmorphism $(z,p,q) \mapsto (z,P,Q)$, preserving z and α (see [1], §1, example 4).

The linear symplectic transformation

$$P_1 = p_1 + aq_2,\ Q_1 = q_1 + bq_2,\ P_2 = p_2 - bp_1 + aq_1,$$
$$Q_2 = q_2, P_i = p_i,\ Q_i = q_i (i > 2)$$

acts in the 3-space with coordinates (p_1, q_1, q_2). The line, dual to the plane $q_2 = 0$ with respect to the nondegenerate form F, is transformed into the q_2 axis (for suitably choosen a and b). Hence the fӧrm F is reduced to $f(p_1, q_1) + cq_2^2$.

A linear symplectic transformation of the plane (p_1, q_1) reduces f to the form $C(p_1^2 \pm q_1^2) + Bq_2^2$. A linear symplectic transformation $Q_2 = Aq_2, P_2 = A^{-1}p_2$ reduces $|B|q_2^2$ to $|C|Q_2^2$ and hence we obtain $H_2 = C(P_1^2 \pm Q_1^2 \pm Q_2^2)$ with independent signs. In the case where H_2 contains $+Q_1^2$, it contains $-Q_2^2$, because H_2 changes the sign. In this case H_2 has the form required in Lemma 3.

In the case where H_2 contains $-Q_1^2 - Q_2^2$ it also has the required form. In the case where it contains $-Q_1^2 + Q_2^2$ we obtain the required form after a linear symplectic transformation $P_1 = q_1, Q_1 = -p_1$.

All the transformations used above are admissibile, hence Lemma 3 is proved. ∎

LEMMA 4_r. (main Lemma) *Let $H = H_2 + R_r$ be an admissible function, where $H_2 = (p_1^2 \pm q_1^2 - q_2^2)/2$ and the remainder $R_r(x), x = (z,p,q)$, has a zero of order* *at least* $r \geq 3$ *at the origin. Then there exist an admissible contactmorphism* g *and a function* E *such that* $E(x)H(g(x)) \equiv H_2(x) + R_{r+1}(x)$, *(i. e. the order of the zero of the remainder at the origin is at least* $r+1$), *while the difference between* g *and the identity has at the origin a zero of order at least* $r-1$ *and the difference between* E *and* 1 *has a zero at the origin of order at least* $r-2$.

The proof of this Lemma is given in §2.

Proof of the theorem. There exist Darboux coordinates, fer which H satisfies the conditions of Lemma 4_3 (by Lemma 3). The admissible contactomorphism g of Lemma 4_3 and the multiplication by E reduce H to a function satisfying the conditions of Lemma 4_4. Using this Lemma, then Lemma 4_5 and so on, we obtain a sequence of admissible contactomorphisms and a sequence of multiplicators, reducing H to H_2 up to terms of higher and higher order. The Taylor series of these contactomorphisms and multiplicators at 0 stabilize (the terms of any given order do not change after some step of the construction), since the order of tangency to the identity and to 1 of the contactomorphism g and of the function E given by Lemma 4_r grow with r. This proves the theorem. ∎

2. THE PROOF OF THE MAIN LEMMA

DEFINITION. *The contact vector field with Hamilton function* K is the field v defined, in the Darboux coordinates (1), by the equation

$$\dot{p} = -K_q + pK_z/2, \dot{q} = K_p + qK_z/2, \tag{2}$$
$$\dot{z} = K - pK_p/2 - qK_q/2$$

(here and below the dot means the derivative along the vector field v). The phase flow of this vector field preserves the contact structure, but not the form $\alpha : L_v\alpha = K_z\alpha$ (where L_v is the Lie derivative).

REMARK. Any vector field, preserving the contact structure, is locally defined by the equation (2) for some Hamilton function K, but we shall not use this fact.

DEFINITION. A contact field is *admissible*, if it is tangent to the manifold of vertices $p_1 = q_1 = q_2 = 0$. The contactmorphism of the phase flow of an admissible vector field preserves the manifold of vertices. Formula (2) implies the

LEMMA 5. *The contact fields with Hamilton functions*

$$K = p_1^2 K_1 + p_1 q_1 K_2 + q_1^2 K_3 +$$
$$+ p_1 q_2 K_4 + q_1 q_2 K_5 + p_2 q_2 K_6 + K_7,$$

where K_7 is inependent of (p_1, q_1, p_2), are admissible. ∎

REMARK. These are all the admissible fields, but we shall not use this fact.

Formula (2) implies also the

LEMMA 6. *Let H be a homogeneous polynomial of degree s in the Darboux coordinates (p, q, z), K – a homogeneous polynomial of degree r. Then the derivative of H along the contact field defined by the Hamiltonian function K is equal to $\dot{H} = \{K, H\} +$ (terms of order higher than $s + r - 2$), where the Poisson bracket $\{K, H\} = K_p H_q - K_q H_p$ is a homogeneous polynomial of degree $s + r - 2$.*

For instance, in the cas $\deg H = 2$ (resp. $\deg H > 2$) $\deg\{K, H\} = \deg K$ (resp. $\deg\{K, H\} > \deg K$).

We shall prove the main Lemma 4 for the case $H_2 = p_1 q_1 - q_2^2/2$. The case $H_2 = p_1^2 - q_1^2 - q_2^2$ is reducible to the preceding one (Lemma 3). In the case $H_2 = p_1^2 + q_1^2 - q_2^2$ the proof is similar (one uses the complex conjugate coordinates $p_1 \pm i q_1$). Below H_2 always means $p_1 q_1 - q_2^2/2$, R_r is thè remainder of a Taylor series, having at the origine a zero of order at least r.

LEMMA 7. *Let $H = H_2 + R_r$ be an admissible function, $r > 2$. Then there exists an admissible contactomorphism g_1, such that the function $H_1(x) \equiv H(g_1(x))$ has the form*

$$H_1(x) \equiv H_2 + h_r(\rho, p_2, Z) + R_{r+1}(x) \tag{3}$$

where $H_2 = \rho - q_2^2/2$, $\rho = p_1 q_1$, $Z = (z, p_3, q_3, \dots, p_D, q_D)$, h_r is a homogeneous polynomial of degree r in the Darboux coordinates (p_1, q_1, p_2, Z), while the difference between g_1 and the identity transformation has at the origin a zero of order at least $r - 1$.

The main ingredient of the proof is the following.

LEMMA 8. *Let F be a homogeneous polynomial of degree r of the Darboux coordinates, which contains no monomials of the form $\rho^a p_2^c Z^M$ (where M is a multiindex). Then the homological equation*

$$\{K, H_2\} = F$$

is solvable with respect to the homogeneous polynomial K of degree r.

If, in addition, F is an admissible polynomial, then there exists a homogeneous solution K, for which the vector field with the Hamilton function K is admissible.

Proof of Lemma 8. Let us introduce the notations:

$$K = \sum K_{a,b,c,d,M} p_1^a\, q_1^b\, p_2^c\, q_2^d\, Z^M,$$
$$F = \sum F_{a,b,c,d,M} p_1^a\, q_1^b\, p_2^c\, q_2^d\, Z^M.$$

The homological equation may be written in the form of a system

$$(a-b)\, K_{a,b,c,d,M} - (c+1)\, K_{a,b,c,+1,d,-1,M} = F_{a,b,c,d,M}$$

of linear equations with respect to the coefficients of the polynomial K. This system decomposes into independent subsystem, corresponding to the fixed values of a, b, M. Every subsystem has the Jordan form, namely

$$(a-b)\, k_d - (c+1)\, k_{d-1} = f_d,\ c+d = r-a-b-|M|, k_{-1} = 0.$$

If $a \neq C$, the determinant of the subsystem does not vanish and hence the subsystem is solvable. If $a = b$ the subsystem is solvable, provided that $f_d = 0$. Hence the homological equation is solvable, provided that F contains no monomials for which $a = b, d = 0$, that is no monomials $\rho^a p_2^c Z_M$. The first statement of the Lemma is proved.

The admissibility condition for a monomial F has the form $a+b+d \geq 2$. Let us consider the 3 cases: 1. $a+b \geq 2$. The monomials of the above constructed solution K of the homological equation with right hand side F, still have $a+b \geq 2$. Hence the contact field with Hamiltonian function K is admissible by Lemma 5 (cases K_1, K_2 and K_3).

2. $a+b=1$. In this case $a \neq b$, $d \geq 1$. The homological equation whose right hand side is a monomial F, divisible by q_2^d, admits a solution K, which is a homogeneous polynomial divisible by q_2^d. This follows from the explicit Jordan structure of the corresponding subsystem (to which we add the conditions $k_{d-1} = k_{d-2} = \ldots = 0$) : its determinant does not vanish, because $a \neq b$.

All the monomials of the solution K defined above are divisible either by $p_1 q_2$ or by $q_1 q_2$. The contact field with Hamilton function K is admissible by Lemma 5 (cases K_4, K_5).

3. $a+b=0$, hence $d \geq 2$. The homological equation with monomial F of this kind in the right hand side has the form $-(c+1)k_{d-1} = f_d$. The solution K, which is a monomial $k_{d-1} p_2^{c+1}\, q_2^{d-1} Z_M$, is divisible by $p_2 q_2$. The contact vector field with Hamilton function K is admissible by Lemma 5 (case K_6).

Lemma 8 is proved. ∎

The proof of Lemma 7. Let h_r denote the sum of the monomials of the form $\rho^a p_2^c Z^M$ of degree r of R_r. The sum of all the other degree r monomials of R_r will be denoted by $-F_r$. By Lemma 8 the homological equation with right hand side F_r has a solution K_r which is a homogeneous polynomial of degree r and which defines an admissible contact vector field.

Denote by g_1 the phase flow transformation of this vector field at time 1. Then, for $H_1(x) \equiv H(g(x))$, we find

$$H_1 = H + \{K_r, H\} + \ldots =$$
$$= H_2 + (h_r - F_r) + \{K_r, H\} + \ldots =$$
$$= H_2 + (h_r - F_r) + F_r + \ldots,$$

as required (the dots are terms of order greater than r by Lemma 6). ∎

LEMMA 9. *Let h be a quasihomogeneous polynomial of weight r in variables $(x, \ldots)$ of weights $(a, \ldots)$. Then*

$$h(x, \ldots) - h(y, \ldots) = (x - y) N(x, y, \ldots)$$

where N is a quasihomogeneous polynomial of weight $r - a$ in variables $(x, y, \ldots)$ of weights $(a, a, \ldots)$.

Proof. $x^n - y^n = (x - y)(x^{n-1} + \ldots + y^{n-1})$ ∎

LEMMA 10. *The function H_1 of Lemma 7 admits a representation*

(4) $$H_1 \equiv H_2 + h_r(q_2^2/2, p_2, Z) + H_1 N_{r-2} + \ldots,$$

(where N is a homogeneous polynomial of degree $r - 2$ and the dots are terms of order $r + 1$ or greater).

Proof. By Lemma 9

$$h_r(\rho, p_2, Z) - h_r(q_2^2, p_2, Z) = (\rho - q_2^2/2) N_{r-2}$$

If we substitute H_1 in place of the multiplicator $\rho - q_2^2/2 = H_2$ in this formula, the product increment will be a small quantity of order al least $2r - 2 \geq r + 1$ at the origine (according to (3)). This implies (4). Lemma 10 is proved. ∎

Let us define $1 - N_{r-2} = E_1$. We obtain from (4)

(5) $$E_1 H_1 \equiv H_2 + h_r(q_2^2/2, p_2, Z) + \ldots$$

(with the dots of order at least $r + 1$).

LEMMA 11. *The homological equation*

$$\{K, H_2\} = -h_r\,(q_2^2/2, p_2, Z)$$

admits a solution K, *which is a homogeneous polynomial of degree* r, *divisible by* $p_2\, q_2$ *and independent of* p_1 *and* q_1.

Proof. The homological equation for K independent of p_1 and q_1 has the form $q_2\, K_{p_2} = h$. The function h is divisible by q_2^2, since the function $E_1\, H_1 = H_2 + h_r + \ldots$ is admissible. The required solution is given by the formula $k_{d-1} = h_d/(c+1)$ in the notations of the proof of Lemma 8. K is divisible by $p_2\, q_2$, because d ≥ 2 since h is divisible by q_2^2. ∎

LEMMA 12. *There exists an admissible contactomorphism* g_2, *reducing the function given by formula* (5) *to the form*

$$E_1(g_2\,(x))H_1\,(g_2\,(x)) \equiv H_2 + R_{r+1} \tag{6}$$

such that the difference between g_2 *and the identity transformation is a small quantity of order at least* $r - 1$ *at the origin.*

Proof. The required contactomorphism g_2 is the transformation of the phase flow of the contact vector field Hamilton function K defined by Lemma 11, corresponding to the time moment $t = 1$.

This contactomorphism is admissible by Lemma 5 (case K_6).

Formula (6) proves Lemma 12 and hence the main lemma:

$$E(x) \equiv E_1(g_2(x)), g(x) \equiv g_1(g_2(x)).$$ ∎

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Manuscript received: July 1, 1988